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The Successes and Failures of the Constituent Quark Model

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I. Introduction - History of the Constituent Quark Model

As I mentioned in my 1978 Banff lectures[1] a nonrelativistic constituent quark model has been remarkably successful in describing many phenomena in hadron physics.[1-7] However there are other areas where the model has been spectacularly unsuccessful.[8-10] George Zweig used to say that the quark model[11,12] gives an excellent description of half the world.

The model has no sound theoretical basis. In the early days there was no clue to the underlying theory. Today we believe that the underlying theory is QCD and that hadrons are composed of quarks and gluons. However the equations of QCD are so complicated that no one has been able to solve them to derive hadron spectroscopy and dynamics. The constituent quark model can now be considered as an intermediate phenomenological model which fits the experimental data and will hopefully be derived from QCD.

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Our approach considers the model as a possible bridge between QCD and the experimental data and examines its predictions to see where these succeed and where they fail. Pinpointing the successes and failures may give us clues to the connection with the underlying theory. We also attempt to improve the model by looking for additional simple assumptions which give better fits to the experimental data. But we avoid complicated models with too many ad hoc assumptions and too many free parameters; these can fit everything but teach us nothing. We also attempt to look beyond the simple ad hoc assumptions to see whether QCD gives any indication for a possible justification.

Everyone has his own version of the quark model. We define our *constituent* quark model by analogy with the *constituent electron model of the* atom and the constituent nucleon model of the nucleus. In the same way that an atom is assumed to consist only of constituent electrons and a central Coulomb field and a nucleus is assumed to consist only of constituent nucleons hadrons are assumed to consist only of their constituent valence quarks with no bag, no glue, no ocean, nor other constituents. Although these constituent models are oversimplified and neglect other constituents we push them as far as we can. Atomic physics has photons and vacuum polarization as well as constituent electrons, but the constituent model is adequate for calculating most features of the spectrum when finer details like the Lamb shift are neglected. Similarly, constituent nucleon models are used extensively in nuclear spectroscopy even though we know that at some stage the contributions of mesons, isobars, exchange currents, etc. must be also taken into account.

The simple nonrelativistic constituent quark model has had remarkable success in describing the low-mass spectroscopy of the quark-antiquark and three quark systems.[1-7] Most recently it has given excellent descriptions of quarkonium systems like charmonium and the T states.[5,6] However for detailed properties of multiquark systems the model has failed almost completely and given no predictions which have been verified by experiment.[8-10] In this talk, we shall try to understand how the model can be so successful in the quark-antiquark and three quark systems and fail for almost everything else.

We first review the history of the constituent quark model. This can be conveniently divided into three stages.

A. The simple nonrelativistic quark model[11,12] (no color) was first introduced to explain the quantum numbers occurring in the low-lying meson and baryon spectrum. This model was then extended to treat all possible properties of hadrons by making the simplest dynamical assumptions possible.[2,3] Additive single quark operators were used wherever possible to describe observable properties of hadrons, including weak,[13] electromagnetic[14,15] and strong[16,17] interaction matrix elements. Nucleon-antinucleon annihilation was described as quark rearrangement.[18] Hadron mass splittings were described by a nuclear shell model approach with two-body interactions.[19,20] The results obtained were in surprising agreement with experiment for all of hadron spectroscopy. However, there were several outstanding open problems.

1. The statistics problem. Although quarks were expected to obey Fermi statistics, the wave functions required to fit the baryon spectrum were symmetric in all the known degrees of freedom rather than antisymmetric.

2. The saturation problem. Quarks had not been observed and were assumed to have a high mass. Mesons and baryons were thus strongly bound systems indicating the presence of strong attractive forces both in the quark-antiquark and quark-quark systems. There was no explanation for why only

quark-antiquark and three quark systems should be strongly bound. This paradox is illustrated most dramatically by comparing the well known deuteron stripping reaction with its analog in hadron physics

$$d + H^3 \rightarrow n + \alpha \tag{1a}$$

$$M + B \not\rightarrow q + \bar{q}qqq \tag{1b}$$

In a collision between a deuteron and a triton, the proton can be stripped from the deuteron and combined with the triton to make an alpha particle with considerable energy release. Since the nucleon-nucleon force is strongly attractive, the proton in the deuteron is much more strongly attracted by the three nucleons in the triton than by the single neutron in the deuteron and the transfer releases energy. In the analogous case of a meson-baryon collision (1b) one would expect the antiquark in the meson to be much more strongly attracted to the three quarks in the baryon than to the single quark in the meson. However the antiquark stripping reaction (1b) does not occur.

The stripping reaction (1b) might somehow be ruled out by a general principle forbidding production of states having fractional charge. But it would not forbid production of multiquark states with integral charges which are also not seen. There is no bound state of two pions with charge 2 and a mass below two pion masses. Such multiquark states with integral charge and baryon number are now called exotics. They might still be found as high mass resonances, but there are clearly no stable low-mass bound states. Their absence could not be explained in any simple way by the naive constituent quark model without the color degree of freedom. If the quark-quark and quark-antiquark forces are attractive in all channels as is implied by the existence of meson and baryon bound states with all possible quantum numbers, then the four-quark state shown in Fig. 1 should be bound much more strongly than two separated quark-antiquark pairs.

3. The meson-baryon problem. There was no simple description of the forces required to make both mesons and baryons. A vector interaction like electrodynamics would give attractive quark-antiquark forces and repulsive quark-quark forces and would not bind baryons. A scalar or tensor interaction would give attractive and equal quark-quark and quark-antiquark forces and lead to a diquark spectrum identical to the meson spectrum. Combinations of vector and scalar or tensor interactions to make the quark-quark force attractive but weaker than the quark-antiquark forces could explain the difference between mesons and baryons. But this seemed contrived and not very convincing.

4. The free quark problem. There was no reasonable explanation for the failure to find free quarks.

The solution of the statistics problem was found[21] in 1964. The additional internal degree of freedom now called color enabled quarks to satisfy Fermi statistics with wave functions symmetric in all the previously known degrees of freedom and antisymmetric in color.

In 1968 some simple features of the $N \to \infty$ limit[22] where N is the number of colors were noted as a possible explanation for how hadrons could be bound states of quarks but free quarks would not be created in hadron-hadron collisions. If quark-antiquark pairs were bound into mesons by an interaction characterized by a coupling constant g, the binding energy of the state we now call the color singlet is proportional to Ng^2. However the quark-quark scattering cross section and the meson-meson scattering cross section would be proportional to g^2 without the factor N. Thus in the limit where $N \to \infty$ but

Ng^2 remains finite quark-antiquark pairs would be bound into mesons but the meson-meson scattering cross section which might break the mesons up into their constituent quarks would go to zero This explanation for the failure to find quarks is not considered seriously today. But the simplification of the large N limit has been rediscovered in the context of QCD where the decoupling of hadron-hadron interactions in this limit makes it a useful starting point for expansions in strong interaction dynamics.

B. The global color nonrelativistic quark model. The color degree of freedom was explicitly introduced into phenomenological dynamical quark models in 1973 to solve the saturation and meson-baryon problems.[23] The use of a nonabelian gauge theory with confinement had not yet proposed. At the 1972 Batavia conference, Murray Gell-Mann in his summary[24] presented a strong case for the color degree of freedom based primarily on the electromagnetic and weak interactions and suggested that some kind of vector gluons were responsible for the strong interactions. However there was no suggestion that the gluons were colored or that the color degree of freedom was in any way essential for the strong interactions. There was no hint of asymptotic freedom, infra-red slavery or nonabelian gauge theories with confinement.

The global color model[23] considered a two-body interaction which would be produced by the exchange of an octet of colored massive vector bosons.[25] The bosons like the quarks would have to be massive to explain the failure to observe them experimentally. Confinement was not understood at that time and the only mechanism to explain the failure to observe such particles was by giving them a high mass. The color-exchange Yukawa interaction solved the saturation and meson-baryon problems. The quark-antiquark and three quark systems behaved like neutral atoms, there were no strongly bound exotics and the quark-quark interaction was exactly half of the quark-antiquark interaction

$$V(qq) = \frac{1}{2} V(q\bar{q}) \tag{2}$$

This is exactly the relation required to bind both mesons and baryons.

In a model where quarks were very massive and hadrons had a very low mass on the quark mass scale, the expression (2) could explain the existence of mesons and baryons by setting the strength of the matrix element of the interaction (2) to be approximately equal to the quark mass. Then

$$\langle V(qq)\rangle \approx \frac{1}{2} \langle V(q\bar{q})\rangle \approx -M_q \tag{3a}$$

$$\langle V(q\bar{q})\rangle \approx -2M_q \tag{3b}$$

$$3 \langle V(qq)\rangle \approx -3M_q \tag{3c}$$

where M_q is the quark mass. The interaction between the two quarks in a diquark thus cancels only one quark mass of the two body system and leaves the diquark with essentially the same high mass as the quark. The quark-antiquark interaction which cancels two quark masses leaves mesons down at low mass and the three quark-quark interactions in a baryon cancels the three quark masses to give low-lying baryons. However there was still no justification for a nonrelativistic picture nor for the use of the interaction (2) which comes from one gluon exchange in a strong interaction model where multiple gluon exchanges are not easily neglected.

The model gave no strongly bound exotic state. With potentials having a reasonable spatial variation, there was no possibility of getting a lower energy than that of two spatially separated mesons. The two-body interactions are attractive and described by the expressions (2) and (3) only in for the color singlet quark-antiquark configuration found in mesons and the color antitriplet quark-quark configuration found in baryons. The interactions in the quark-antiquark color octet configuration and in the quark-quark color sextet configuration are both repulsive. These repulsive channels play no role in meson and baryon states but are responsible for the saturation in the multiquark states. The attractions and repulsions cancel in calculating the force between a color singlet hadron and an external quark in the same way that the Coulomb attractions and repulsions cancel in the force from a neutral atom on an external charged particle.

C. The QCD motivated nonrelativistic quark model. After the introduction of asymptotic freedom and confinement in nonabelian gauge theories led to the development of QCD,[4,26] De Rujula, Georgi and Glashow[27] introduced ideas from QCD into the nonrelativistic quark model. They attributed the spin dependence of the two-body interaction to the spin dependent part of a one gluon exchange interaction. This explained for the first time the sign of the hyperfine splittings; e.g. why the Δ is heavier than the nucleon, and related the magnitudes of the hyperfine splittings to the quark masses. With this model it was possible to obtain two independent relations between the strange and nonstrange quark masses in terms of experimental hadron masses[27,28] and to use them to predict the Λ magnetic moment.[1,27,29] Both of these predictions agreed exactly with one another and with the experimental value of the Λ moment. The original prediction by DGG in 1975 is particularly impressive because it was made before the magnetic moment had been measured precisely.

In the multiquark sector, this model gave complete nonsense. The saturation feature of the global color model remained. Forces between color singlet hadrons were much weaker than those that bind the quarks into hadrons. But the introduction of confining potentials led to unphysical long range forces which were in disagreement with experiment[8-10] and rapidly changing color[30] correlations between spatially separated systems which violated causality. Multiquark baryonium states were predicted and not found experimentally but created considerable confusion as one candidate after another was shown to be only a statistical fluctuation.[31]

The situation can be summed up by saying that the nonrelativistic constituent quark model with input from QCD gives a very good phenomenological description of the quark-antiquark and three quark systems but breaks down in the multiquark sector. Arguments explaining this breakdown are presented below in Section IV.

In Section II we examine the possible basis from QCD of one application of the simple additive quark model with single-quark operators, the calculation of hadron total cross sections. In Section III we look beyond the approximation of single quark operators for phenomenological contributions from two-quark operators in total cross sections and magnetic moments. In Section IV we analyze the difference between the quark-antiquark and three quark systems where the model succeeds and the multiquark systems where the model fails and show how gluon dynamics can make the difference. Somehow it is possible to replace the gluon field by an effective interaction in the quark-antiquark and three quark systems. But the gluon dynamics plays an essential role in the multiquark system and the constituent quark picture is no longer adequate.

II. Why are Total Cross Sections Additive?

The additive quark model (AQM) for high energy scattering[17] has been remarkably successful in fitting and predicting experimental data, despite the absence of a satisfactory QCD derivation. It was first introduced on the basis of an impulse approximation as shown in Fig. 2. This ad hoc assumption was never justified but accepted as reasonable. However it appeared to be completely wrong after the advent of QCD. Single gluon exchange between separated color-singlet hadrons cannot occur. Two-gluon exchange shown in Figs. 3a and 3b is a two-quark operator which violates the impulse approximation of the AQM when the two gluons are emitted by two different quarks in the same hadron as in Fig. 3b. A two-gluon exchange model for the pomeron contribution to scattering proposed independently by Nussinov[32] and Low[33] explained some dynamic features, but did not relate meson and baryon couplings in a simple way. However it turns out that the combinatorial factors in two-gluon and three-gluon exchange vertices do indeed satisfy the additivity assumption of the AQM in lowest order, even though there is no impulse approximation and two-quark and three-quark operators play an important role.[34,35] The color algebra allows the multiquark operators to be reduced to single quark operators and gives the simple quark-counting rules relating meson and baryon couplings. The problems never solved in the original impulse approximation approach remain; e.g. ignoring the differences between mesons and baryons in form factors, masses and momentum fractions carried by an active quark. However the additional difficulty of obviously wrong combinatorial factors arising from two-quark and three-quark operators is resolved.

The quark counting factor was first noted by Gunion and Soper.[34] We follow the general algebraic derivation of Ref. 35 which obtains the result explicitly from the color algebra of the vertices describing gluon emission from color singlet hadrons shown in Figs. 4a and 4b. These vertices have a very simple color structure because of the factorization of the color degree of freedom which allows all color factors to be expressed in terms of generators of the SU(3) color group.

The emission of a gluon by a single quark is described in the quark space by the matrix element of a color octet operator between two color triplet states. By the Wigner-Eckart theorem, this matrix element factorizes into the product of a reduced matrix element of a color-independent operator and an SU(3) Clebsch-Gordan coefficient which is the same for any color octet operator and is therefore proportional to the matrix element of the corresponding generator of SU(3) color. Thus we can write

$$\langle (G^a) q_i' | V | q_i \rangle = \langle q_i' \| A_i \| q_i \rangle \langle q_i' | F_i^a | q_i \rangle \qquad (4a)$$

where F_i^a denotes the eight SU(3) color generators for the ith quark of antiquark and a is a color index, G^a denotes a gluon with the color quantum number a, q_i and q_i' denote any two states of the i-th quark or antiquark and the double-barred reduced matrix element of the color-independent operator A_i describes all the other degrees of freedom except color. The generator F_i^a is identical to the λ^a matrix[4,8,26,36] for quark states. For antiquarks $F^a = (-\lambda^a)^*$.

Color factorization also occurs in the meson and baryon wave functions with a color-independent factor in the baryon wave function totally symmetric under permutations of the quarks. The reduced matrix elements of products of two color-independent single quark operators A_i and B_j are thus independent of the quark indices i and j.

$$\langle H\| A_i B_j \| H\rangle = M_1 \delta_{ij} + M_2 - M_1 \tag{4b}$$

where M_1 and M_2 are independent of the indices $\underline{i}$ and $\underline{j}$. The matrix element (4b) has the value either M_1 or M_2 depending only upon whether $\underline{i}$ and $\underline{j}$ are the same or different, but not on their explicit values.

Consider the vertex $\langle(GG)H|V|H\rangle$ describing the emission from a color singlet hadron H of two gluons in an overall color singlet state. Two types of contributions arise. The gluons can be emitted either by the same quark in the hadron H as in Fig. 4a or by two different quarks as in Fig. 4b.

$$\langle(GG)H|V|H\rangle = \sum_a \left\{ f_1 \sum_i \langle H|F_i^a F_i^a|H\rangle + f_2 \sum_{j\neq i} \sum_i \langle H|F_i^a F_j^a|H\rangle \right\} . \tag{5}$$

The coefficients f_1 and f_2 denote all the additional dynamical and kinematic factors which do not depend upon the color degree of freedom obtained by evaluating matrix elements of color-independent operators.

The second term on the right hand side of eq. (5) is manifestly non-additive and depends quadratically on the quark number. However, a hidden $\underline{n}$-dependence in the color coupling factors exactly cancels the non-additive combinatorial factor to give an overall result proportional to the quark number. This can be shown by using the following identities for color couplings with any color singlet quark-antiquark or three-quark hadron $|H\rangle$.

$$\sum_i F_i^a |H\rangle = 0 \tag{6a}$$

$$\sum_a F_i^a F_i^a |H\rangle = c|H\rangle \tag{6b}$$

where c is the eigenvalue of the quadratic Casimir operator for SU(3) color for a single quark state and n_H is the number of constituents in hadron $\underline{H}$; i.e. $n_H = 2$ for mesons and 3 for baryons. We first rewrite eq. (5) as

$$\langle (GG)H|V|H\rangle = \sum_a (f_1-f_2) \sum_i \langle H|F_i^a F_i^a|H\rangle + f_2 \sum_j \sum_i \langle H|F_i^a F_j^a|H\rangle). \tag{7}$$

The two-body terms in eq. (7) can be seen to vanish by the identity (6a) that the sum of any SU(3) color generator F_i^a over all quarks is a generator of the total SU(3) color group which annihilates any color singlet state. The remaining term is evaluated with the identity (6b). Thus

$$\langle (GG)H|V|H\rangle = \sum_a (f_1-f_2) \sum_i \langle H|F_i^a F_i^a|H\rangle = (f_1-f_2)n_H c \ . \tag{8}$$

This result (8) shows that the two-gluon emission vertex for any hadron H are proportional to the quark number n_H. A similar result was obtained for the three-gluon vertex. Any model for Pomeron exchange such as that of Ref. 2 in which the coupling of the Pomeron to the hadron $\underline{H}$ is via two or three gluons will thus give the AQM ratio of 3/2 for baryons to mesons for the imaginary part of the forward scattering amplitude and the total cross section. This includes not only the contributions from the simple gluon-exchange diagrams but also from more complicated ladder exchanges which couple to the external hadrons via a two-gluon or three-gluon vertex.

The inclusion of four-gluon exchanges breaks the additive quark model. That additivity cannot be saved by manipulation of color factors is easily seen in the example of one possible four-gluon diagram with two gluons coupled to a color singlet and coupled to a single quark and the reamining two gluons coupled to any two quarks and coupled to a color singlet as in the two gluon vertex (5). This contribution then factorizes into two two-gluon vertices, one behaving like the first term on the right hand side of eq. (5) and the second like the entire right hand side. The result is a quadratic function of quark numbers.

III. Beyond the Additive Quark Model

The assumption that all possible processes are described by single-quark operators must break down at some level. The question arises whether it makes sense at all to attempt to describe higher order effects by two-quark operators, or whether the whole model should be discarded at the 15% level where additivity breaks down. This question was investigated[37] in 1974 with the aim of looking for some signal in the discrepancies at the 10% level between AQM predictions and experimental data on total cross sections which were sufficiently precise to look at 1% physics. Since Regge exchange was the popular mechanism for high energy scattering at that time, the dominant mechanism described by a flavor-dependent two-quark operator was a double exchange involving a pomeron exchange and an exchange of the f-meson Regge trajectory. The simplest test of this model gave new relations between hadron total cross sections which were in remarkable agreement with experiment. The situation was characterized by the following remark by one of my colleagues: "I do not believe a word of this crazy model. But the numbers are impressive. You must find a better explanation." For eight years a better explanation has been sought but not found. Instead all that has been found are more and more impressive numbers, showing agreement with new data on hadron nucleon total cross sections at higher energies and new channels and on real parts of forward amplitudes with the simple predictions of this "two-component Pomeron model" which adds a second component described by a two-body operator to the simple quark-counting first component.[37]

The basic feature of the data described by this model is the simultaneously breaking of quark model additivity and SU(3) symmetry. These two effects are then seen to be empirically related and both described by a single mechanism. The additive component of the total cross section due to Pomeron exchange was assumed to be universal and a pure SU(3) singlet with no symmetry breaking. All the strangeness dependence of the Pomeron component as well as the meson-baryon difference came from a single non-additive second component which enhanced contributions from nonstrange quarks by an amount depending upon the total number of quarks in the hadron. The SU(3) breaking appeared as an enhancement of the contribution from the nonstrange quarks, rather than as a suppresssion of the contribution of the strange quarks.

The most recent success of this model is in the hyperon-nucleon cross sections[38] which had not been measured when the model was first proposed. In the AQM this dependence is universal in mesons and baryons and attributed to the difference at the quark level between the scattering amplitudes of the strange and nonstrange quarks. The difference between $\sigma(\Xi N)$, $\sigma(\Sigma N)$ and $\sigma(NN)$ must then be equal to the difference between $\sigma(KN)$ and $\sigma(\pi N)$.[39]

$$\sigma(pp) - \sigma(\Sigma p) = \sigma(\Sigma p) - \sigma(\Xi p) = \sigma(\pi^- p) - \sigma(K^- p). \qquad (9a)$$

The two-component pomeron model, on the other hand, attributes the strangeness dependence to a second order effect which is a quadratic function of the quark numbers, rather than a linear function. The change in total cross section when a nonstrange quark is replaced by a strange quark is larger in baryons than in mesons by a factor of 3/2.[40,41]

$$\sigma(pp) - \sigma(\Sigma p) = \sigma(\Sigma p) - \sigma(\Xi p) = (3/2)\{\sigma(\pi^- p) - \sigma(K^- p)\}. \qquad (9b)$$

This difference by a factor of 3/2 between the predictions (9a) and (9b) of the AQM and of the two-component pomeron model has now been tested experimentally. The prediction (9b) from the two-component pomeron model agrees with experiment.

The general approach of this two-component pomeron model pinpoints certain features of the experimental data which have a simple physical interpretation. This is most clearly demonstrated by inverting the relations between hadron nucleon and quark nucleon amplitudes and obtaining the contributions of strange and nonstrange quarks to the experimental baryon-nucleon and meson-nucleon cross sections.

$$\sigma(nN)_B = (1/6)\{\sigma(pp) + \sigma(pn)\} \qquad (10a)$$

$$\sigma(nN)_M = (1/4)\{\sigma(\pi^- p) - \sigma(K^- p) + \sigma(\pi^+ p) - \sigma(K^- n) + \sigma(K^+ p) + \sigma(K^+ n)\} \qquad (10b)$$

$$\sigma(sN)_B = (1/6)\{\sigma(\Sigma^- p) + \sigma(\Sigma^- n) + \sigma(\Xi^- p) + \sigma(\Xi^- n) - \sigma(pp) - \sigma(pn)\} \qquad (10c)$$

$$\sigma(sN)_M = (1/4)\{\sigma(K^- p) - \sigma(\pi^- p) + \sigma(K^- n) - \sigma(\pi^+ p) + \sigma(K^+ p) + \sigma(K^+ n)\} \qquad (10d)$$

where $\sigma(nN)_B$, $\sigma(nN)_M$, $\sigma(sN)_B$ and $\sigma(sN)_M$ denote the contributions from nonstrange and strange quarks to the isospin averaged baryon-nucleon and meson-nucleon scattering cross sections respectively as calculated from the

AQM and the conventional duality assumption of equality of the contributions from strange quarks and antiquarks is used to eliminate antiquark contributions from eqs. (10b) and (10d).

$$\sigma(sN)_M = \sigma(\bar{s}N)_M \tag{10e}$$

The AQM predicts the equality of the corresponding quark-nucleon contributions to baryon and meson cross sections. Substituting the relations (10) gives two sum rules which can be tested against experimental data:

The nonstrange sum rule,

$$\sigma(nN)_B = \sigma(nN)_M \tag{11a}$$

$$\begin{aligned}(1/6)\{\sigma(pp)+\sigma(pn)\} &= \\ &= (1/4)\{\sigma(\pi^- p)-\sigma(K^- p)+\sigma(\pi^+ p)-\sigma(K^- n)+\sigma(K^+ p)+\sigma(K^+ n)\}\end{aligned} \tag{11b}$$

$$12.9 \pm 0.01 \text{ mb.} = 11.2 \pm 0.05 \text{ mb.} \tag{11c}$$

and the strange sum rule,

$$\sigma(sN)_B = \sigma(sN)_M \tag{12a}$$

$$\begin{aligned}(1/6)\{\sigma(\Sigma^- p)+\sigma(\Sigma^- n)+\sigma(\Xi^- p)+\sigma(\Xi^- n)-\sigma(pp)\sigma(pn)\} &= \\ &= (1/4)\{\sigma(K^- p)-\sigma(\pi^- p)+\sigma(K^- n)-\sigma(\pi^+ p)+\sigma(K^+ p)+\sigma(K^+ n)\}\end{aligned} \tag{12b}$$

$$7.7 \pm 0.1 \text{ mb.} = 7.75 \pm 0.05 \text{ mb.} \tag{12c}$$

The experimental data quoted are taken at 100 Gev/c momentum, where there are both new data on hyperon-nucleon cross sections and previous data on the other hadronic cross sections available.

The strange sum rule (12) is seen to be in excellent agreement with experiment, while there is strong disagreement with the nonstrange sum rule (11). The 15% discrepancy is significant and shows that the contribution from strange quarks to the hadron-nucleon cross sections is the same in mesons and baryons but that the contribution from nonstrange quarks is greater in baryons than in mesons. This indication that strange quark contriutions are somehow simpler than nonstrange contributions is a significant and recurrent feature of the data which has no explanation from first principles. The model also relates the SU(3) symmetry breaking which produces the difference between the strange and nonstrange contributions (11) and (12) to the breaking of aditivity in the sum rule (11). The assumption that single two-quark mechanism explains both effects gives a new relation between these quantities

$$\sigma(nN)_B - \sigma(sN)_B = (3/2)\{\sigma(nN)_M - \sigma(sN)_M\} \tag{13a}$$

$$(1/3)\{\sigma(pp) + \sigma(pn)\} - (1/6)\{\sigma(\Sigma^- p) + \sigma(\Sigma^- n) + \sigma(\Xi^- p) + \sigma(\Xi^- n\} = \\ = (3/4)\{\sigma(\pi^- p) - \sigma(K^- p) + \sigma(\pi^+ p) - \sigma(K^- n)\} \tag{13b}$$

$$5.15 \pm 0.07 \text{ mb.} = 5.2 \pm 0.1 \text{ mb.} \tag{13c}$$

Here the SU(3) symmetry breakings in the baryon and meson sectors are compared and shown to be related in the exact number predicted by the two-component pomeron model. This prediction has now been strikingly confirmed by the new hyperon-nucleon data. It is just the factor 3/2 appearing in eqs. (13a) and (9b) that makes the difference between the two predictions (9a) and

(9b). The analogous sum rule from the AQM differs from (6a) by not having the factor of 3/2 on the right hand side. The value of 3.46 ± 0.08 mb. obtained without the factor 3/2 is in strong disagreement with the value 5.15 ± 0.07 mb. on the left hand side.

The two-component pomeron model was extremely successful in fitting data available at the time and has successfully predicted a large quantity of data from subsequent experiments. The striking agreement between prediction and experiment shown in eqs. (12-13) is particularly impressive since no data was available at this energy and no hyperon-nucleon cross sections had been measured at all when the model was first proposed. The success of these sum rules seems to indicate that the breaking of SU(3) and additivity are mysteriously related and that the corrections to the simple SU(3)-symmetric AQM affect only the contributions of the nonstrange quarks.

The model has no convincing derivation from first principles. The original double exchange picture fails to explain the observed energy dependence, which differs from predictions from pomeron-f double exchange. The success of the sum rules has motivated a search for an alternative mechanism to give a contribution with same dependence on quantum numbers as pomeron-f exchange but a different energy dependence. So far this search has been unsuccessful.

At this point one might look for further clues in other properties of hadrons where the AQM breaks down. Baryon magnetic moments have been treated with an additive quark model where SU(3) breaking is introduced by suppression the additive contribution of the strange quarks.[14,42] However, it now appears that additivity is also broken.[43] It may be that here also the SU(3) breaking mechanism is better described as a non-additive enhancement of the nonstrange quark contribution, rather than a suppression of the additive strange quark contribution. We therefore examine the present status of baryon magnetic moments from this point of view.

The essential details of the problem are shown in Table 1.

TABLE I

Theoretical and Experimental Values of Baryon Magnetic Moments

Baryon	SU(3) Symmetric Model	Experimental Value[44,45]	Standard Broken SU(3)	Two Comp. Broken SU(3)
proton	1.83	2.793	2.79	2.75
neutron	-1.22	-1.913	-1.86	-1.83
Λ	-0.61	-0.614±0.005	-0.61	-0.61
Σ^+	1.83	2.33±0.13	2.67	2.24
Σ^-	-0.61	-0.89±0.14	-1.09	-0.81
Ξ^-	-0.61	-0.75±0.06	-0.50	-0.61
Ξ°	-1.22	-1.25±0.014	-1.44	-1.22
$R(p,\Sigma^+,\Xi^-)$	0	2.76±0.85	0.38	2.50
$R(\Xi,\Lambda)$	1.0	1.09±0.03	1.06	1.0
$R'(\Xi,\Lambda)$	1.0	1.12±0.02	1.0	1.0
$R''(\Xi,\Lambda)$	1.0	1.22±0.1	0.82	1.0

Included in the table are four functions of the magnetic moments which project out certain physically interesting features of the data.

$$R(p,\Sigma^+,\Xi^-) = -3\{\mu(p) - \mu(\Sigma^+)\}/\{\mu(\Xi^0) - \mu(\Xi^-)\} \tag{14a}$$

$$R(\Xi,\Lambda) = \{\mu(\Xi^0) + \mu(\Xi^-)\}/3\mu(\Lambda) \tag{14b}$$

$$R'(\Xi,\Lambda) = \{\mu(\Xi^0) + 2\mu(\Xi^-)\}/4\mu(\Lambda) \tag{14c}$$

$$R''(\Xi,\Lambda) = \mu(\Xi^-)/\mu(\Lambda) \tag{14d}$$

The expressions (14a) and (14b) were defined in Ref. 43. The expressions (14c) and (14d) are modified versions of (8b) which are somewhat more sensitive to disagreements wtih the model predictions.[46]

Motivated by the suggestion that the AQM works better for strange quarks we begin our analysis from the unorthodox SU(3) symmetry limit in which all quarks have the mass of the strange quark and use the Λ magnetic moment as input. The magnetic moment of a baryon with a configuration denoted by (1,2;3), where quarks 1 and 2 have the same flavor, is then given in the static model with L=0 SU(6) wave functions by

$$\tilde{\mu}(1,2;3) = (2q_1 + 2q_2 - q_3)\,\mu(\Lambda) = (4q_1 - q_3)\mu(\Lambda), \qquad (15a)$$

where $\tilde{\mu}$ denotes the magnetic moment in the SU(3) limit where all quarks have the mass of the strange quark, q_i denotes the electric charge of quark i, and $q_1 = q_2$. This SU(3)-symmetric form is scaled to give the correct Λ moment.

The predictions of the standard broken-SU(3) model are obtained from this limit by enhancing the contributions of the nonstrange quarks by a factor which fits the proton moment, while leaving the strange quark contributions unchanged.

$$\mu(1,2;3) = (2q_1+2q_2-q_3)\mu(\Lambda) + (2q_1x_1+2q_2x_2-q_3x_3)\{(1/3)\mu(p)-\mu(\Lambda)\}$$

$$= (4q_1-q_3)\mu(\Lambda) + (4q_1x_1-q_3x_3)\{(1/3)\mu(p)-\mu(\Lambda)\} \qquad (15b)$$

where x_i is defined to be 1 if quark i is a nonstrange quark and zero if it is a strange quark and $x_1 = x_2$. The symmetry breaking terms are all proportional to x_i which vanishes for strange quarks. Thus the scaling of the strange quark contribution to the moment remains unchanged. Equation (15b) can be rewritten

$$\mu(1,2;3) = (2q_1+2q_2-q_3)(1/3)\mu(p) - \\ - \{2q_1(1-x_1) + 2q_2(1-x_2) - q_3(1-x_3)\}\{1/3)\mu(p)-\mu(\Lambda)\} \\ = (4q_1-q_3)(1/3)\mu(p)-\{4q_1(1-x_1) - q_3(1-x_3)\}\{1/3)\mu(p)-\mu(\Lambda)\} \quad (15c)$$

This is the conventional form in which the SU(3) symmetry breaking appears as a suppression of the strange quark contribution rather than an enhancement of the nonstrange contribution. The SU(3)-symmetric term is scaled to give the correct proton moment and the symmetry breaking terms are all proportional to $(1-x_1)$ which vanishes for nonstrange quarks.

If the comparatively small discrepancy in the Ξ^- moment is neglected, the predictions in the SU(3) symmetry limit shown in Table I are seen to fit the Ξ moments reasonably well and the nucleon moments very badly. The introduction of symmetry breaking fits the nucleons well, but at the price of a much stronger disagreement in the Ξ sector. The Σ^+ and Σ^- moments are midway between the two predictions, but the large error on the Σ^- moment allows a fit to either within two standard deviations. Furthermore, the discrepancy in the difference between the proton and Σ^+ moment remains and is nearly unaffected by the symmetry breaking.

This paradox suggests that SU(3) symmetry breaking is not a simple phenomenon and goes beyond enhancing the magnetic moment of the nonstrange quarks relative to that of the strange quarks by the same factor in all hadrons. *Nonstrange quark moments seem to be quenched in strange hadrons*[43] (or equivalently enhanced in nonstrange hadrons), perhaps by pion exchange.[47,48] However, the results tabulated in Table I show a very large enhancement of about 50% needed to fit the nucleon data, with no enhancement required for the Ξ's. This suggests that all SU(3) breaking comes from a new dynamical mechanism like pion exchange, described by a two-body operator enhancing the nonstrange contributions mainly in the nucleon, with no additional breaking from the quark mass difference. This is difficult to believe; yet it corresponds exactly to the model described above for high energy scattering. The common feature in the total cross section and magnetic moment data that the nonstrange quarks break the SU(3) symmetry rather than the strange quarks may be significant.

This point motivated a "two-component" model[46] with one fully SU(3)-symmetric component given by eq. (9a) and the second component breaking SU(3) with a non-additive enhancement of the contributions of the nonstrange quarks by an ad hoc factor chosen to fit the nucleon moments $1+(1/4)N$, where N is the number of additional nonstrange quarks in the baryon; i.e. $N = 0,1$ and 2 in the Ξ, Σ and nucleon respectively.

$$\begin{aligned}\mu(1,2;3) &= (2q_1+2q_2-q_3)\mu(\Lambda) + \\ &+ \{2q_1x_1(x_2+x_3) + 2q_2x_2(x_3+x_1)-q_3x_3(x_1+x_2)\}\{\mu(\Lambda)/4\} \\ &= (4q_1-q_3)\mu(\Lambda) + \{2q_1x_1(x_1+x_3)-q_3x_3x_1\}\{\mu(\Lambda)/2\} \qquad (16)\end{aligned}$$

The simultaneous breaking of SU(3) symmetry and additivity is evident in this relation, since the symmetry-breaking terms all contain two-quark products x_ix_j. This contrasts with eq. (9b) where all symmetry breaking terms are linear in the x_i's. The predictions of this model are listed in Table I as "Two Comp. Broken SU(3)."

The empirical formula (16) has no theoretical basis. Its superiority over the standard model which also has only one SU(3)-breaking parameter shows that the data are parametrized better by non-additive rather than additive enhancement of the nonstrange quark contribution. The factor (1/4) is chosen to fit the well-known approximate enhancement factor of 3/2 for the nucleon. This corresponds in the standard model to a quark mass ratio of 3/2. With parameters fixed in both models by fitting the nucleon and Λ, the significant test comes in the Σ moments. The additive standard model (9b) predicts the same nonstrange enhancement in all baryons and gives the same enhancement in the Σ as in the nucleon in disagreement with experiment. The data show that the nonstrange enhancement needed for the Σ is roughly half that needed for the nucleon, which agrees with the prediction of the non-additive two-component model, eq. (16).

Models for hadron structure and dynamics must eventually describe all properties including masses, magnetic moments and scattering cross sections. The standard broken-SU(3) model predicts the Λ magnetic moment from the proton moment and hadron mass differences. This clue to hadron structure should not be easily discarded in correcting the model to fit other hyperon moments. It is interesting and puzzling that both the magnetic moments and the total cross sections are fit reasonably well by a two-component description in which both additivity and SU(3) symmetry are broken only by a second component which enhances nonstrange contributions. Unfortunately there is no simple relation between the two "second components" in Eqs. (13) and (16). Further experimental work in hyperon physics may help to clarify these paradoxes. A model which explains why strange quarks seem to have a simpler behavior than nonstrange quarks would be very interesting.

IV. Problems of the Constituent Quark Model for Multiquark States

Models with only quark degrees of freedom and effective interactions are analogous to conventional atomic models with only electrons and nuclei and a Coulomb interaction. But the gluon of non-Abelian QCD with its color charge and nonlinear self coupling is very different from the neutral photon. The color current carried by the gluon field plays an essential role in ensuring local conservation of color charge and local gauge invariance.

The charge of an electron cannot be changed by photon emission nor shielded by the surrounding cloud of virtual photons. Charge exchange does not occur between electrons and nuclei in atoms. Maxwell's equations are linear and the lines of force from a point charge radiate istropically in all directions as shown in Fig. 5a. An additive static Coulomb interaction proportional to gauge-invariant c-number charges and uniquely determined by the positions and charges of all particles is adequate for atomic physics. The contribution from the Coulomb field of a nucleus to the force on an atomic electron is independent of the positions of the other electrons. All effects of screening of the nuclear coulomb field are included by simply adding the coulomb fields of all charged particles.

The color charge of a quark is changed by gluon emission and shielded by virtual gluons. Color exchange occurs in interacting multiquark systems and depends upon gluon dynamics. The color charge of a quark is a gauge-dependent dynamical variable subject to quantum fluctuations. A quark-antiquark pair at two different space points cannot be in a color singlet state in all gauges. A local gauge transformation at the position of the quark changes its color without changing the color of the antiquark. The QCD field equations are nonlinear and the lines of color force are confined to strings or flux tubes as shown in Fig. 5b. The forces on a quark due to its interactions with two other quarks at different space points are not additive, because of the nonlinear couplings in the gluon field between them. The color energy of a given configuration is not completely determined by the locations and color charges of the constituent quarks. It also depends upon the configuration of the strings or flux tubes connecting these constituents.

Consider a multiquark system containing two quarks and an antiquark as shown in Figs. 6a and 6b. A flux tube may connect the antiquark with either quark while another flux tube connects the remaining quark to the rest of the system. The force on the antiquark depends not only on the positions of all other constituents, but also on whether its flux tube connects it to one quark or the other as shown in Figs. 6. The dynamics of the gluon field may cause the flux tube to jump back and forth between the two configurations. An adequate description of these dynamics may not be possible in a model with only quarks and effective static interactions and no flux tubes. Greenberg and Hieterinta[49] have argued that the string degrees of freedom must be included in any phenomenological model describing the long range forces in multiquark systems.

A potential model with colored quarks interacting via a phenomenological color-exchange force motivated by one-gluon-exchange was first introduced before QCD to explain the saturation of interquark forces in hadrons and the relation between meson and baryon spectra.[23] Only global color symmetry was assumed, with massive gluons giving a short-range Yukawa interaction and no gauge theory nor confinement. This model was later connected with QCD by hand-waving arguments[10] without noting the inconsistency introduced by the additional requirement of local gauge invariance and color charge conservation in a constituent colored quark model with no explicit description of the gluon degrees of freedom.

This inconsistency can be seen[9] by applying a local gauge transformation to a color-singlet quark-antiquark wave function,

$$|1\rangle = \sum_{a} \bar{q}_a(x)q_a(y) \tag{17}$$

where _x_ and _y_ are two different space points and _a_ is a color index. The state $|1\rangle$ is manifestly a color singlet under a global SU(3) color transformation.

We now apply a _local_ SU(3) color gauge transformation which acts only at the point _y_ and not at the point _x_ and is described by a unitary matrix U_{ab} in color space chosen to have all its diagonal elements vanish,

$$|1\rangle \rightarrow \sum_{ab} U_{ab}\bar{q}_a(x)q_b(y) = |8\rangle \tag{18a}$$

where

$$U_{aa} = 0 \quad . \tag{18b}$$

The state $|8\rangle$ is a pure octet under global SU(3) color and has no singlet component. Since all physical hadron states in QCD are assumed to be color singlets and color octet states do not exist, an allowed local guage transformation (18) which transforms the color singlet state $|1\rangle$ into the color octet state $|8\rangle$ indicates an inconsistency with local gauge invariance.

One might try to avoid this inconsistency by choosing a particular gauge before using the wave function (17). However the following physical argument suggests that the inconsistency results from the neglect of the role of the gluon current in ensuring local conservation of color and probably cannot be eliminated by choosing an appropriate gauge. The global color singlet wave function $|1\rangle$ has a quark and an antiquark at two different space points with instantaneously correlated color fluctuations. The quark and antiquark each have equal probabilities of being red, blue or green, but the antiquark must be green when the quark is green. In a complete QCD description color is locally conserved and is exchanged between the quark and antiquark only via gluon exchange. The wave function $|1\rangle$ must have a "string" of gluon current connecting the points $\underline{x}$ and $\underline{y}$ to conserve color and preserve local gauge invariance. This difficulty does not arise in abelian QED with neutral photons where electric charge is trivially conserved in photon exchange.

We can now see the implications of this inconsistency for the colored constituent quark model which disregards the contribution of the gluon field to the color current. The model fails consistently in cases where localized color densities occur in the quark sector and gluon currents necessary for current conservation must introduce additional dynamic effects like screening. All phenomenological successes of the model occur when color and space are separated; the failures occur when they are not. The model succeeds in the quark-antiquark and three-quark systems where the meson and baryon wave functions factorize into a color factor and factor depending upon all other degrees of freedom. Color completely separates from space and all dynamics are described by a color-independent effective interaction. The color degree of freedom enters only in two ways:

1. The allowed states for three-quark baryons are required to be <u>symmetric</u> in the other degrees of freedom in accordance with Fermi statistics for colored quarks in a color singlet state.[21]

2. A color factor related to the eigenvalue of a color SU(3) Casimir operator appears in relating quark-quark interactions in baryons to quark-antiquark interactions in mesons.[23]

These successes suggest that whenever color and space factorize the inconsistency (18) is unimportant and the dynamics of the gluonic degrees of freedom can either be neglected or somehow absorbed into an effective constituent quark wave function without gluons. This occurs rigorously when

$\underline{x} = \underline{y}$ and there is no string or flux tube because all the quarks are at the same point. The transformation (16) is then not allowed and the state $|1\rangle$ remains a color singlet under all gauge transformations. The effective constituent quark picture might still hold for a wave function which includes terms with $\underline{x} = \underline{y}$ and where color and space factorize, so that all the color transformation properties are determined by the portion of the wave function with $\underline{x} = \underline{y}$ and there are no correlations between color and space.

The model has failed when wave functions without factorization of color and space are used to describe multiquark systems and correlation between color and space play an essential role; e.g. in "color chemistry" predictions of unobserved "baryonium" states with color-space correlations[31] and the calculation of long range Van der Waals forces between separated hadrons[1,8,10] arising from the admixture into the two-hadron wave function of states describing spatially separated color-octet pairs coupled to an overall color singlet. Such correlations are not gauge invariant in a model which considers only quark degrees of freedom. In QCD with confinement any local color-octet density in the quark sector is screened by gluons and the oversimplified constituent quark picture which does not include gluon dynamics cannot be valid.

The predictions of exotic multiquark hadrons bound by color hyperfine interactions[49,50] with color-spin correlations but no color-space correlations have not yet conclusively succeeded or failed. No such multiquark states have yet been convincingly identified, and further experimental tests are of great interest.[51] The four-quark model for scalar mesons[49-52] bound by hyperfine interactions explains properties of the low-lying scalar mesons δ and S^* which are otherwise very mysterious. A zero-range hyperfine interaction which acts only between a pair of quarks at the same point produces bound states analogous to the deuteron with two quark-antiquark clusters separated by a distance of several cluster radii.[52] These states can be described as two color-singlet clusters, bound by an effective short-range potential obtained from a hyperfine interaction, with no unphysical color-space correlations.

This picture might be test by experiments producing the δ and S^* on nuclear targets. The reaction

$$K^- + p \rightarrow \Lambda + M^0 \tag{19}$$

where M° is a neutral meson should have an A dependence when the proton is bound in a nucleus which depends on the <u>size</u> of the meson M°. If the δ and S^* are large four-quark states they should be absorbed much more strongly in a nuclear target than $q\bar{q}$ states like ρ^0, ω, ϕ, f, A2 and f' which should be copiously produced in the same reactions. Studying the $\eta\pi$ spectrum in such reactions would enable direct comparison of the A-dependence of δ and A2 production, while the $\pi\pi$ spectrum would compare S^* production with ρ° and f° production.

One might even expect that stripping reactions analogous to (1) would occur between a deuteron-like S^* and δ and a nucleus

$$(\delta^0, S^*) + (Z,A) \rightarrow {}_\Lambda(Z,A) + K^0 \tag{20a}$$

$$(\delta^0, S^*) + (Z,A) \rightarrow {}_\Lambda(Z-1,A) + K^+ \tag{20b}$$

where ${}_\Lambda(Z,A)$ denotes the hypernucleus with charge Z and baryon number A. Comparing the reactions (19) and (20) might lead to production of hypernuclei with double strangeness.

$$K^- + (Z,A) \rightarrow (\delta^0, S^*) + {}_\Lambda(Z-1,A) \rightarrow {}_{\Lambda\Lambda}(Z-1,A) + K^0 \qquad (21a)$$

$$K^- + (Z,A) \rightarrow (\delta^0, S^*) + {}_\Lambda(Z-1,A) \rightarrow {}_{\Lambda\Lambda}(Z-2,A) + K^+ \ . \qquad (21b)$$

We now use the string picture to see the role of gluon dynamics in more detail. The original global SU(3) potential model[23] takes a two-body potential, introduces a color factor appropriate for a one-gluon-exchange Yukawa interaction and applies it everywhere to multiquark systems. This is clearly unjustified in a gauge theory. However the potential model has a number of interesting properties, of which some can be expected to hold in a correct description and others to break down completely.

The potential used has the form[8]

$$H_I = -\sum_{i>j} F_i \cdot F_j \, V(|x_i - x_j|) \qquad (22)$$

where F_i denotes the eight SU(3) color generators for the ith quark or antiquark.

In the color-singlet quark-antiquark state used in meson spectroscopy the color coupling of a quark and an antiquark to a color singlet is unique and factors out. The interaction (22) can be made identical (by hand) to the credible effective two-body potential used in charmonium obtained from QCD considerations such as lattice gauge calculations. This picture has a string between the quark and antiquark as shown in Fig. 7a and the effective potential is obtained by summation over all string configurations.

In the three-quark color singlet state used in baryon spectroscopy, described in Fig. 7b the color coupling is also unique and factors out. The interaction given by Eq. (22) for this case has the correct asymptotic form for the case where two quarks are very close together as shown in Fig. 7c. The force between a color triplet quark and a point-like color antitriplet diquark is the same as the quark-antiquark force as expected from QCD. Thus this interaction may be taken as a crude approximation to the correct QCD description of baryon spectroscopy.

For multiquark systems containing more than three constituents the asymptotic behavior of the interaction (22) for states describable as two separated point-like clusters agrees with the expectations from QCD. There is no strong force between two point-like color singlet clusters and the triplet-antitriplet force is identical to the quark-antiquark force. For more complicated spatial configurations the color couplings for an overall singlet state are no longer unique and factorization of color no longer occurs. The interaction (22) is a nontrivial matrix in color space[1,23] with matrix elements depending upon the spatial degrees of freedom. It is no longer positive definite and can produce pathologies with confining potentials which always have "anticonfining" components.[8] Furthermore there is not even a one-to-one correspondence between the configurations allowed by QCD and those allowed by the model based on the interaction (22).

The difficulty can be seen explicitly by examining the system of two quarks and two antiquarks located at the four corners of a tetrahedron. Some possible color couplings for the four particles are:

$$|A\rangle = |\{(13)_1;(24)_1\}_1\rangle \tag{23a}$$

$$|B\rangle = |\{(13)_8;(24)_8\}_1\rangle \tag{23b}$$

$$|C\rangle = |\{(14)_1;(23)_1\}_1\rangle \tag{23c}$$

$$|D\rangle = |\{(14)_8;(23)_8\}_1\rangle \tag{23d}$$

$$|E\rangle = |\{(12)_{3*};(34)_{3}\}_{1}\rangle \tag{23e}$$

$$|F\rangle = |\{(12)_{6};(34)_{6*}\}_{1}\rangle \tag{23f}$$

where particles 1 and 2 are quarks and 3 and 4 are antiquarks and $|\{(ij)_n;(km)_{n*}\}_1\rangle$ denotes a color singlet state with particles $\underline{i}$ and $\underline{j}$ coupled to the representation $\underline{n}$ of SU(3) color and particles $\underline{k}$ and $\underline{m}$ coupled to the representation $\underline{n}^*$.

In a description with strings or flux tubes each of the six configurations (23) is described by drawing strings connecting the four particles in different ways. Figs. 8a, 8b and 8c show the configurations for the states $|A\rangle$, $|C\rangle$ and $|E\rangle$ respectively. The wave function $|\{(ij)_n;(km)_{n*}\}_1\rangle$ denotes configurations with strings joining the pairs of particles (ij) and (km) and additional strings joining these two strings when the pairs (ij) and (km) are not color singlets. Each of these configurations is linearly independent of the other five. This can be seen, for example, by noting that the state $|A\rangle$ described by Fig. 8a has flux tubes along the edges of the tetrahedron joining particles 1 and 3 and joining particles 2 and 4 and no flux elsewhere, while the four states (4c-4f) have no flux along the edges (13) and (24) and all their flux tubes elsewhere. Thus the state $|A\rangle$ cannot be expressed as a liner combination of these four states. The quark description with the interaction (22) has only two independent color couplings.[1,23] The states (23) are linearly dependent in this description and span a Hilbert space of only two dimensions in the color degree of freedom. The state $|A\rangle$ for example is a linear combination of the states $|C\rangle$ and $|D\rangle$ or of the states $|E\rangle$ and $|F\rangle$. Thus there is no simple way to eliminate the QCD strings to obtain an effective interaction of the form (3). Essential information is lost in using a two-dimensional Hilbert space for string dynamics whose description require a space of at least six dimensions.

The quark-antiquark and three-quark systems have unique color couplings and simple flux tube configurations shown in Figs. 7a and 7b, which can probably be replaced by an effective potential. Other multiquark systems no longer have a unique color coupling and several flux tube configurations are equally probable as shown in Figs. 8. For these cases the dynamics of flux tube jumping between these configurations cannot be properly included in a constituent quark description.[9]

In limiting cases where one string configuration is dominant the interaction (22) might describe average static properties with the information about the locations of strings contained in the color coupling factors. However, for calculating forces between hadrons or properties of multiquark matter many string configurations of equal importance arise and any realistic wave function must describe the quantum fluctuations in which strings jump from one configuration to another. There does not seem to be any hope of describing this physics with an interaction of the type (22) or any other model (e.g. bag models) which does not explicitly introduce the dynamics and topology of flux tubes or strings.

Another view of this difficulty is seen from comparison with the Abelian case. The interaction (22) also describes the four-body system of two protons and two electrons if F_i is the electric charge of particle $\underline{i}$ and $V(|x_i - x_j|)$ is the coulomb interaction. If particles 1 and 2 are protons and particles 3 and 4 are electrons, the interaction (22) between proton 1 and electron 3 is always the same coulomb interaction independent of the positions of the other particles and of whether or not proton 1 and electron 4 are bound in a hydrogen atom. The coulomb fields of the four particles are simply additive.

In the nonabelian case this additivity of the two-body forces appears in the interaction (22) but not in QCD. The force between quark 1 and and antiquark 3 as given by interactions (3) depends only upon the positions and color couplings of the two quarks and is independent of the positions of the other particles. But in QCD the force also depends upon how the strings are drawn to the other particles.

It is also instructive to examine the four-body system for the case of a confining potential, such as a harmonic oscillator

$$V(x_i-x_j) = V_o(x_i-x_j)^2 \quad . \tag{24a}$$

Then

$$H_I = \sum_{i>j} \sum_a F_i^a \cdot F_j^a \, V_o(x_i-x_j)^2 =$$

$$-\frac{1}{2} \sum_z \sum_{ij} F_i^a \cdot F_j^a \, V_o(x_i^2+x_j^2) + V_o \sum_a \sum_i F_i^a \vec{x}_i \cdot \sum_j F_j^a \vec{x}_j \quad . \tag{24b}$$

For any color singlet state (or neutral state in the abelian case) the first term on the right hand side of Eq. (24b) vanishes, by virtue of the identity (6a) for any color singlet hadron.

For the two-proton-two-electron system discussed above, with only one value of a and F_i^a denoting the electric charge of i, the interaction (24b) is seen to depend only on the distance between the center of mass of the two protons and the center of mass of the two electrons. Moving the two protons to infinity in opposite directions does not change the interaction if the center of mass of the two is held fixed. Thus although the interaction (24) is clearly confining for the two-body system, the confinement is lost completely in the abelian case for the four-body system. Furthermore, there are extremely pathological long-range forces. A bound two-body system (a harmonic hydrogen atom) on the earth can be broken up by moving the other proton and electron even when they are very far away; e.g. beyond the moon.

This peculiar behavior results from the fact that the interaction (24) is confining only between a pair of particles of opposite charge; i.e. for a quark-antiquark pair in a color singlet state or a quark-quark pair in a color antitriplet state. For other charge or color configurations, the expression is not positive definite and the potential is "anticonfining" and becomes negatively infinite at large distances. The total interaction is seen from Eq. (24b) to be positive definite for color singlet states and unbounded from below for all other configurations.[8]

This discussion reveals an important difference between single-hadron and multiquark states. The interaction (22) seems reasonable for meson spectroscopy, even though its remarkable success in nonrelativistic light quark spectroscopy still remains to be explained. For multiquark spectroscopy the interaction (22) correctly describes the absence of strong forces between separated color singlet clusters[23] but is clearly inadequate for finer details like the treatment of hadron-hadron interactions,[36] multiquark bound states with nontrivial spatial dependences[31] and quark matter.[53] The dynamics of the gluonic degrees of freedom and the gluon color current must be introduced explicitly to describe screening phenomena and keep local color conservation and gauge invariance.[54]

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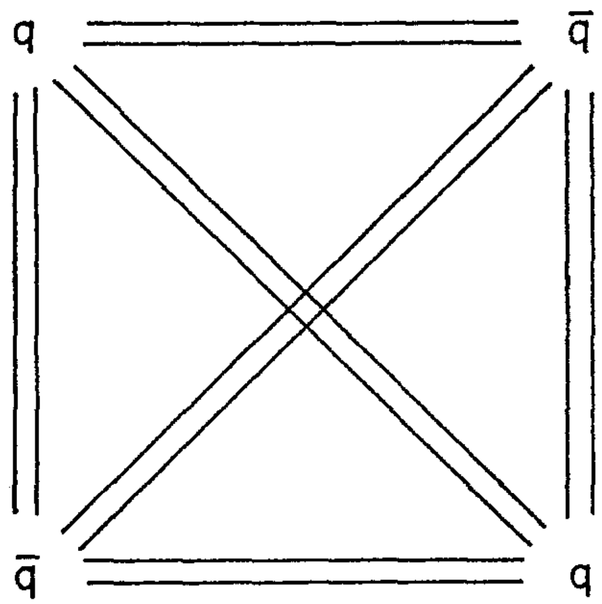


Fig. 1

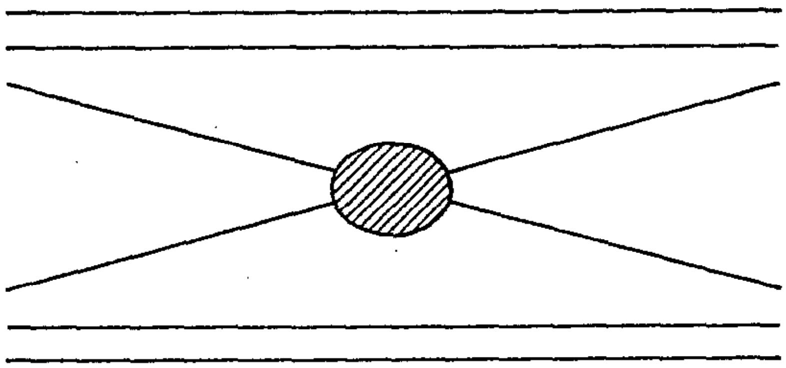


Fig. 2

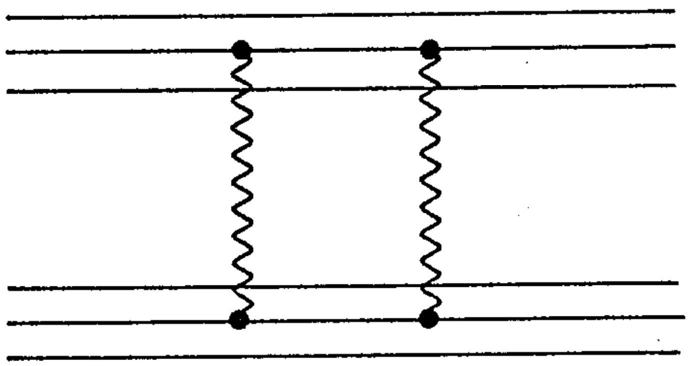
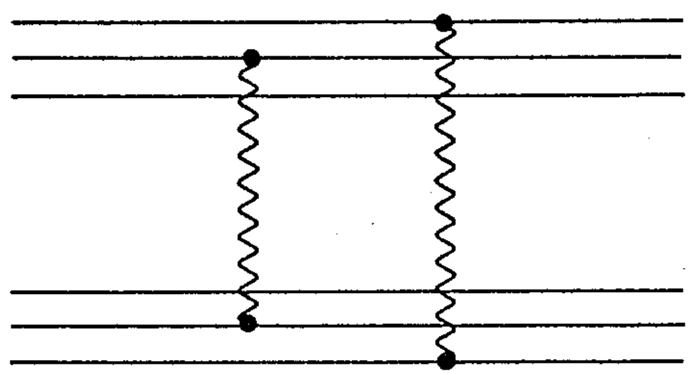


Fig. 3a



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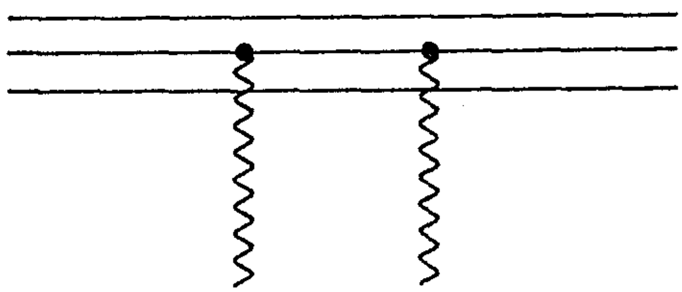

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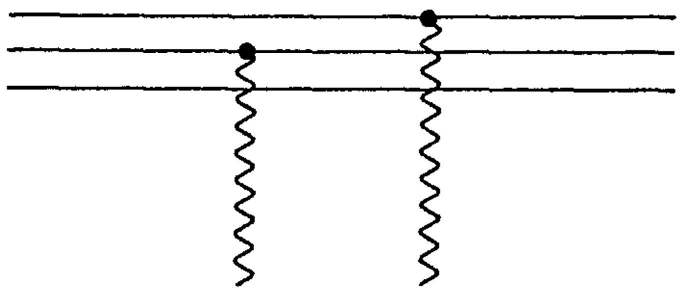

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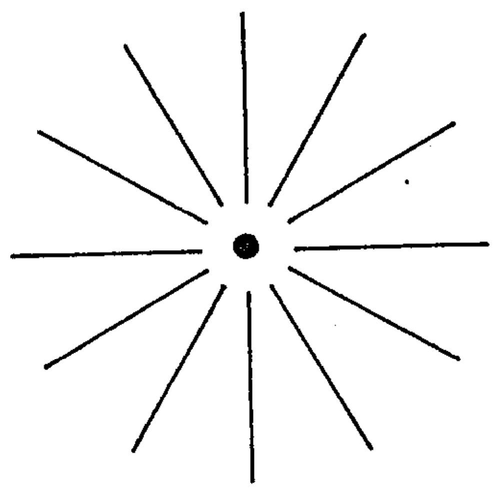
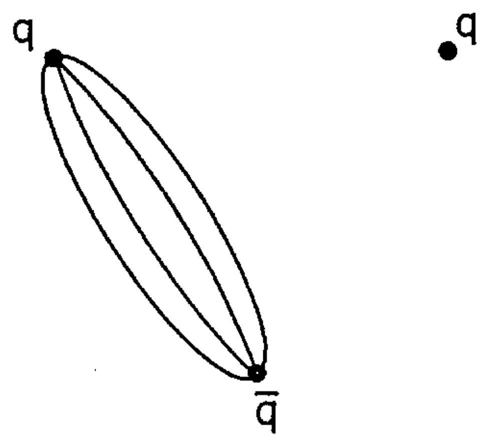


Fig. 5a

Fig. 5b



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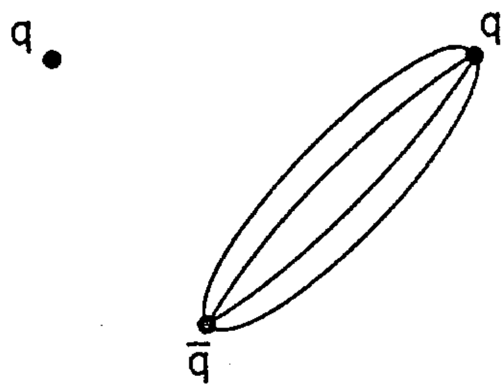
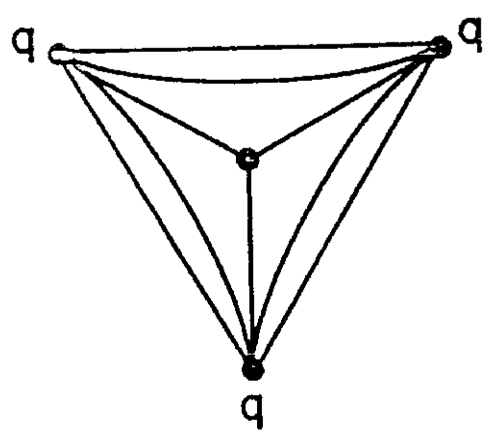


Fig. 6b

Fig. 7a



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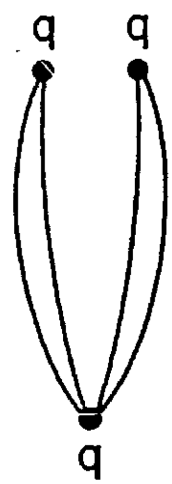

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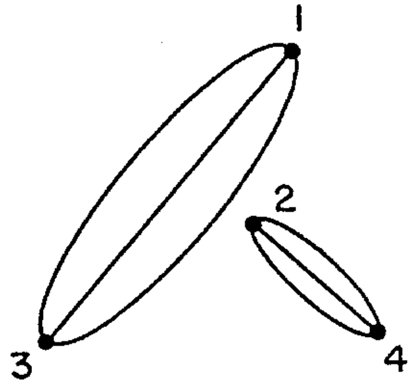

Fig. 8a

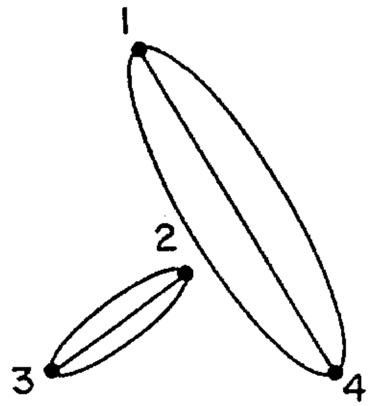

Fig. 8b

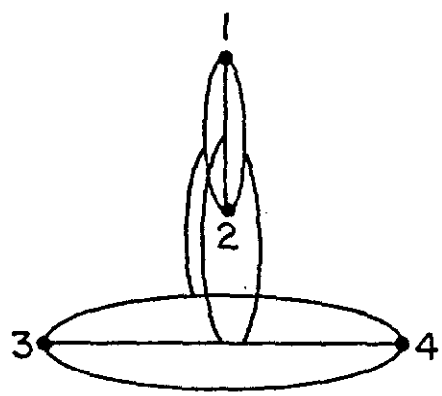

Fig. 8c